Predicting Default with Firm-Specific Macroeconomic Exposures

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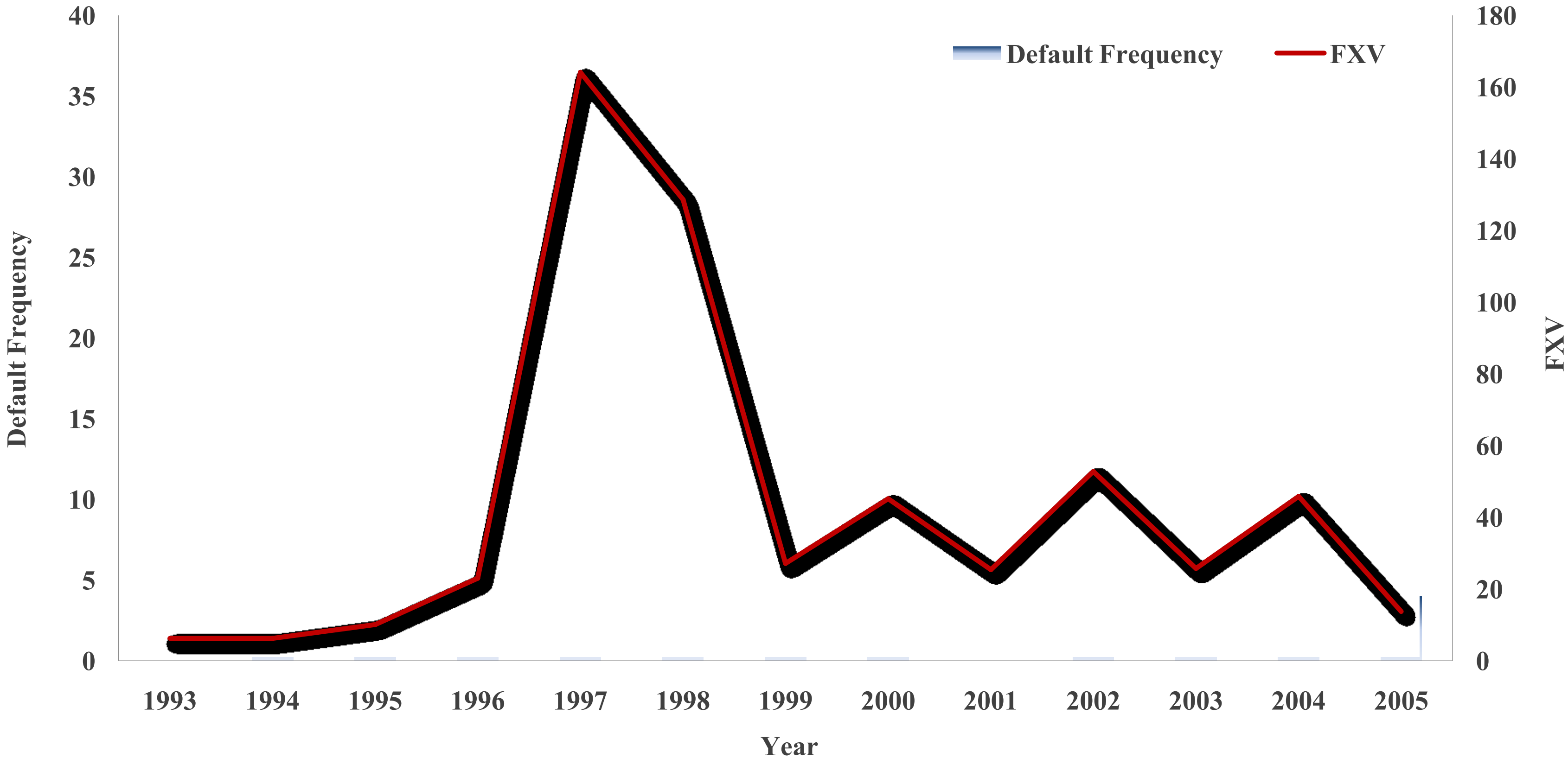
Macro Effect on Defaults



Motivation

- Default likelihood varies with macroeconomic conditions, and certain firms are possibly more affected by macro shocks than others.
- Therefore, it is desirable that default likelihood shows firm-specific sensitivities to changes in macro variables.
- However, current models for default prediction do not capture such heterogeneity.

Literature

- Shumway (2001) introduces a hazard model with time-varying explanatory variables and shows that the model is equivalent to multi-period logit model.

- Chava-Jarrow (2004), Duffie *et al*. (2007), and Nam *et al*. (2008) incorporate industry and macro variables, but with common exposures on common factors.

Default Time

- Default time is modeled as a random variable τ with
  - Probability density function $f(t)$
  - Distribution function $F(t) = \Pr[\tau \leq t]$
  - Survival function $S(t) = \Pr[\tau > t] = 1 - F(t)$

Hazard Rate

- Hazard rate is defined as

$$\lambda(t) = \lim_{dt \to 0} \frac{\Pr[t < \tau \leq t + dt \mid \tau > t]}{dt}$$

- For small dt, λdt approximates the conditional probability that default occurs in (t, $t+dt$) given that the firm survives till t.

- So hazard rate can be considered an instantaneous default rate.

Hazard Rate & Survival Function

- Alternative formula

$$\lambda(t) = \frac{\lim_{dt \to 0} Pr[t < \tau \le t + dt] / dt}{Pr[\tau > t]} = \frac{f(t)}{S(t)}$$

$$\lambda(t) = -\frac{d}{dt} ln\, S(t)$$

- From this, $S(t) = \exp\left\{-\int_0^t \lambda(s) ds\right\}$

  So hazard rate and survival probability have the same information.
- Hence, we can directly model hazard rates.

Proportional Hazard Model by Cox (1972)

- For firm i with covariate x_i,

$$\lambda_i(t; x_i) = \lambda_0(t) exp\{\beta \cdot x_i\}$$

  - For a hypothetical firm with $x = 0$, the hazard rate is $\lambda_0(t)$, which is called the baseline hazard rate.
  - $\exp\{\beta' x_i\}$ gives the relative risk for individual firm characteristics x_i.
  - x_i can be allowed to be time-varying.

Current Default Prediction Models

- Generally, the hazard rate for firm *i* is modeled as

$$\lambda_{it}\left(t; x_i(t), z(t)\right) = \exp\left(\alpha + \beta_x x_i(t) + \beta_z z(t)\right)$$

  - Here x_i is a vector of firm characteristics; and z is a vector of common variables;
  - α, β_x and β_z are constants, which are common to all firms.
  - Heterogeneity is only captured by firm characteristics x_i.
  - This represents the models in Shumway (2001), Chava-Jarrow (2004), Duffie *et al*. (2007), and Nam *et al*. (2008).

How to Obtain Firm-Specific Exposures?

- As discussed, we cannot simply allow β_z to be firm-specific.

- Need to assume a certain structure for β_z

Modeling Macroeconomic Effect

$$\lambda_i\left(t \mid x_i(t)\right) = \lambda_0(t) exp\left\{\beta' x_i(t)\right\}$$

- If we put macro variables into covariate x_i, we cannot have individual sensitivity on the macro factors.

- As the beta coefficients are common to all firms, we can treat betas as factors and consider firm-specific covariates x_i to be factor loadings.

Modeling Macroeconomic Effect

- Let F_j's are macro factors.
- Linear factor structure for beta:

$$\beta_k(t) = b_{k0} + \sum_{j=1}^{M} b_{kj} F_j(t)$$

- Linear factor structure for baseline hazard rate:

$$\lambda_0(t) = exp\left\{ a_0 + \sum_{j=1}^{M} a_j F_j(t) \right\}$$

Modeling Macroeconomic Effect

- With the factor structures, we have the following multi-factor hazard model with firm-specific factor loadings:

$$\lambda_0(t) exp\left\{\beta(t)' x_i(t)\right\}$$
$$= exp\left\{a_0 + \sum_{k=1}^{L} b_{k0} x_{ik}(t) + \sum_{j=1}^{M}\left[a_j + \left(\sum_{k=1}^{L} b_{kj} x_{ik}(t)\right)\right] F_j(t)\right\}$$

Drawing the Analogy

- Assume β_z is explained by firm characteristics x_i.
- The hazard rate can be specified as

$$\lambda_{it}\left(t; x_i(t), z(t)\right) = \exp\left(\left(bx_i(t)\right)^T z(t)\right)$$

where

$$b = \begin{pmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{pmatrix} \quad x_i(t) = \begin{pmatrix} 1 \\ x_{i2}(t) \\ \vdots \\ x_{in}(t) \end{pmatrix} \quad z(t) = \begin{pmatrix} 1 \\ z_2(t) \\ \vdots \\ z_m(t) \end{pmatrix}$$

Example

- If we set $b = \begin{pmatrix} b_{11} & b_{12} & 0 \\ b_{21} & 0 & b_{23} \end{pmatrix} \quad x = \begin{pmatrix} 1 & x_2 & x_3 \end{pmatrix}^T \quad z = \begin{pmatrix} 1 \\ z \end{pmatrix}$

  then $\lambda(t; x, z) = \exp\left(\left(b_{11} + b_{12}x_2\right) + \left(b_{21} + b_{23}x_3\right)z\right)$

  - We can have different sets of explanatory variables for different factors.
  - Previous hazard models are all nested in this model.
  - In essence, firm-specific exposures are obtained by the cross-product terms of firm variables and macro variables.

Data

- Data includes all the firms that have ever listed in KSE and KOSDAQ from 1993 to 2005, excluding financial companies.
- Default event includes bankruptcy filings, workouts and failure to pays.
  - 154 default events out of 1,712 firms.
- For the variable that can represent macroeconomic state of Korea, we consider principal components estimated from FXV, KOSPI return, GDP growth rate and Call rate.

Summary Statistics

Number of Firms

	1993-2000	2001-2005	Total
Non-defaulted	1,444	1,553	1,558
Defaulted	109	45	154

Category	Variable	Definition	Expected Sign	Mean	Median	Std. Dev	Min	Max
Growth	TEGR	total equity growth rate	-	0.4722	0.1196	1.4373	-1.4368	10.6476
Profitability	FCTD	financial costs to total debt	+	0.0513	0.0490	0.0334	0.0000	0.1393
Solvency/Liquidity	CDTA	current debt to total assets	+	0.3821	0.3695	0.1900	0.0342	0.9765
Asset management /Turnover	STA	asset turnover ratio(sales to total assets)	-	1.1615	1.0171	0.6942	0.0883	4.2228
Cash Flow	OCFTA	operating cash flow to total assets	-	0.0276	0.0015	0.1100	-0.4097	0.3341
Market Value	SIGMA	volatility of monthly stock returns	+	0.0363	0.0331	0.0184	0.0014	0.0906
Macroeconomic	METL	market equity to total liabilities	-	1.8490	0.6810	3.6449	0.0240	24.9423
Macroeconomic	FXV	volatility of foreign exchange rate	+	43.6824	25.6800	43.8284	6.2200	164.0700
Macroeconomic	CALL	Call rate	+	7.6491	4.9867	4.3050	3.3308	14.9833
Macroeconomic	GDPGR	GDP growth rate	-	8.3416	7.5500	4.5686	-1.4300	17.2300
Macroeconomic	KOSPIRN	KOSPI return	-	0.1380	0.1861	0.3818	-0.5092	0.8278

Model Specification

- Model I
  - Simple hazard model with only firm characteristics as explanatory variables.

$$\lambda_i\left(t \mid x_i(t-1), F_j(t-1)\right)$$

$$= \exp\left\{a_0 + \sum_{k=1}^{L} b_{k0} x_k(t-1)\right\}$$

- Model II
  - Add a macro variable to Model I. Macro exposure is common to all firms.

$$= \exp\left\{a_0 + \sum_{j=1}^{M} a_j F_j(t-1) + \sum_{k=1}^{L} b_{k0} x_k(t-1)\right\}$$

- Model III (the main model)
  - Add cross-product terms between the macro and firm variables.

$$= \exp\left\{\begin{array}{l} a_0 + \sum_{j=1}^{M} a_j F_j(t-1) + \sum_{k=1}^{L} b_{k0} x_k(t-1) \\ \qquad + \sum_{j=1}^{M} \sum_{k=1}^{L} b_{kj} x_{ik}(t-1) F_j(t-1) \end{array}\right\}$$

Macroeconomic variables

Category	Macroeconomic variables
Output	GDP growth rate, Industrial production, Business inventories
Employment	Unemplyment rate
Inflation	CPI, PPI, GDP Deflator
Fixed-Income Market	Call Rate, CD91, Gov3Y, Slope(Gov5Y - Gov1Y)
Stock Market	KOSPI return, KOSPI vol
Exchange rate	FXV

Univariate tests

Category	Variable	Definition	Expected Sign	Estimate	Std. Err	Wald. Chi.Sq ($\chi2$)
Growth	TAGR	total assets growth rate	-	-0.7297 ***	0.2784	6.8702
	CAGR	current assets growth rate	-	-0.4509 **	0.1950	5.3445
	IAGR	investements assets growth rate	-	-0.2309 **	0.1130	4.1715
	WCGR	working capital growth rate	-	0.0009	0.0395	0.0005
	● TEGR	total equity growth rate	-	-1.3275 ***	0.1774	55.9850
	SGR	sales growth rate	-	-0.6869 ***	0.2647	6.7333
Profitability	ITRR	internal reserve rate	-	-2.4442 ***	0.2395	104.1172
	GPS	gross profit to sales	-	-2.9906 ***	0.8470	12.4673
	OIS	operating income to sales	-	-1.0735 ***	0.3778	8.0759
	ORIS	ordinary income to sales	-	-1.0067 ***	0.1858	29.3442
	NIS	net income to sales	-	-0.9196 ***	0.1830	25.2563
	CGSS	costs of good sold to sales	+	2.9906 ***	0.8470	12.4673
	FCS	financial costs to sales	+	16.3279 ***	1.2438	172.3259
	● FCTD	financial costs to total debt	+	35.1175 ***	3.4467	103.8095
	FCTC	financial costs to total costs	+	26.1846 ***	2.0552	162.3319
	CACL	current ratio(current assets to current liabilities)	-	-0.6306 ***	0.1730	13.2933
	QACL	quick ratio(quick assets to current liabilities)	-	-0.7618 ***	0.2059	13.6827
Solvency/Liquidity	TDTE	total debt to total equity	+	0.1119 ***	0.0157	50.5248
	CSTA	cash to total assets	-	-1.7184	1.3161	1.7047
	● CDTA	current debt to total assets	+	3.5757 ***	0.4913	52.9682
	TBEB	total borrowings to EBITDA	+	0.0085	0.0083	1.0368
	WCTA	working capital to total assets	-	1.1992 **	0.5942	4.0726

Univariate tests

Asset management /Turnover	● STA	asset turnover ratio(sales to total assets)	-	-1.8935 ***	0.2563	54.5593
	STE	equity turnover ratio(sales to total equity)	-	0.0142	0.0172	0.6879
Cash Flow	OCFTD	operating cash flow to total debt	-	-1.3130 ***	0.3557	13.6221
	● OCFTA	operating cash flow to total assets	-	-4.8902 ***	0.8903	30.1730
	OCFS	operating cash flow to sales	-	-2.2332 ***	0.4718	22.4057
	SCFTB	surplus cash flow to total borrowings	-	-0.7531 ***	0.1445	27.1668
	SCFS	surplus cash flow to sales	-	-1.7450 ***	0.5019	12.0895
Market Value	● SIGMA	Sdt.dev of 3 prior month's daily stock returns	+	8.4838 ***	2.6464	10.2771
	● METL	market equity to total liabilities	-	-3.0659 ***	0.4656	43.3558
Macroeconomic	● FXV	volatility of foreign exchange rate	+	0.0097 ***	0.0015	41.9488
	CD	CD(certificates of deposit) rate	+	0.0547 *	0.0330	2.7396
	● CALL	Call rate	+	0.0770 ***	0.0296	6.7413
	● GDPGR	GDP growth rate	-	-0.1274 ***	0.0161	62.5288
	● KOSPIRN	KOSPI return	-	-1.0519 ***	0.2439	18.5947
	PPC	Producer price change	+	0.0388 *	0.0228	2.8818
	CPC	Consumer price change	+	0.0364	0.0475	0.5878
	GDPDP	GDP deflator change	+	-0.0291	0.0333	0.7643

Correlations between macroeconomic variables

	Consumer Price Change	GDP Deflator Change	KOSPIRN	GDPGR	FXV	CALL
Producer Price Change	0.7295 ***	0.4248	-0.0880	-0.7278 ***	0.5085 *	0.5497 *
Consumer Price Change		0.8404 ***	-0.0729	-0.5650 **	0.2753	0.7835 ***
GDP Deflator Change			-0.2030	-0.1313	0.0120	0.7815 ***
KOSPIRN				-0.3057	-0.2617	-0.2337
GDPGR					-0.5572 **	-0.2321
FXV						0.3590

Principal Component analysis

	Eigenvectors			
	Prin1	Prin2	Prin3	Prin4
KOSPIRN	-0.1525	-0.8021	0.3262	0.4763
GDPGR	-0.5454	0.5289	0.1096	0.6409
CALL	0.4989	0.2734	0.8203	0.0586
FXV	0.6560	0.0452	-0.4569	0.5991
Eigenvalue	1.7971	1.2960	0.6761	0.2308
% (Cumulative)	44.9%	77.3%	94.2%	100%

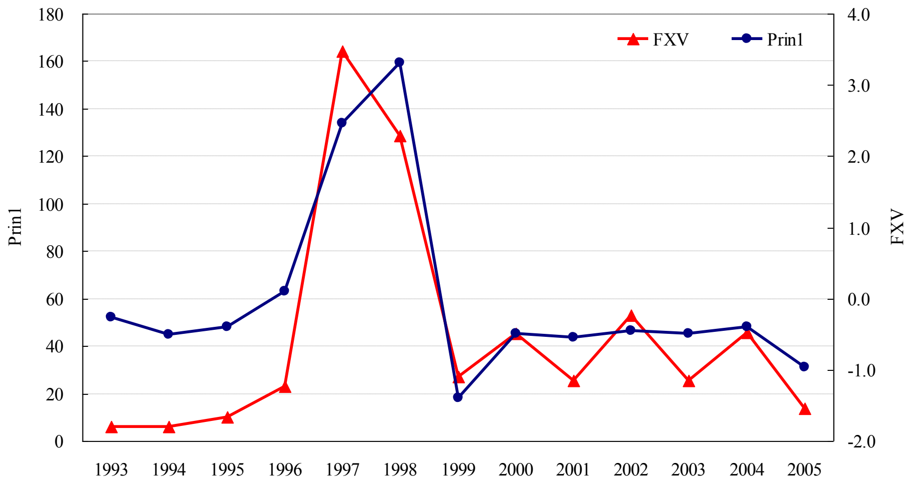
$$Prin1 = -0.1525KOSPIRN_Z - 0.5454GDPGR_Z + 0.4989CALL_Z + 0.6560FXV_Z$$

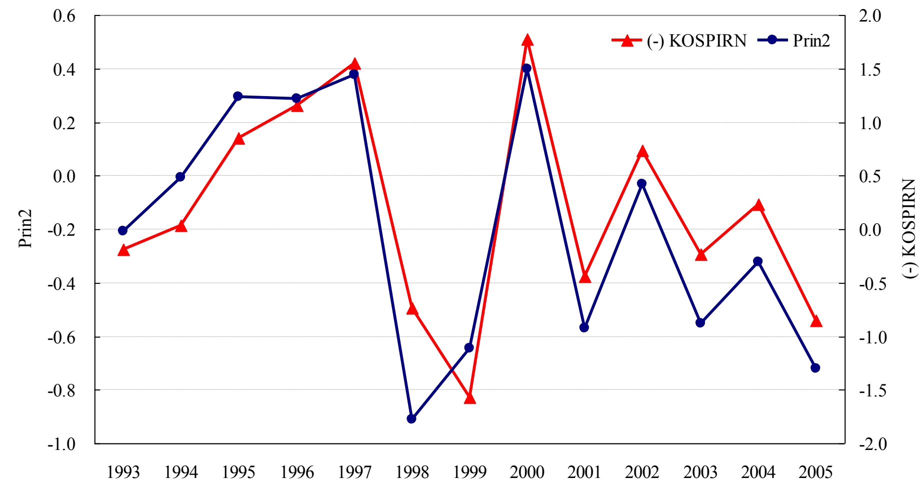
$$Prin2 = -0.8021KOSPIRN_Z + 0.5289GDPGR_Z + 0.2734CALL_Z + 0.0452FXV_Z$$

Ang and Piazzesi (2003) use principal components of macroeconomic variables to obtain the factors to explain term structure movements in the U.S. Treasury market.

Correlations between macroeconomic variables and principal components

	KOSPIRN	GDPGR	FXV	CALL
Prin1	-0.2045	-0.7311 ***	0.8794 ***	0.6689 **
Prin2	-0.9132 ***	0.6021 **	0.0515	0.3113





Estimation Results

	Model I			Model II			Model III		
	Coeff.	χ^2	p-Value	Coeff.	χ^2	p-Value	Coeff.	χ^2	p-Value
Intercept	-5.7396	(92.71)	<.0001	-7.0093	(111.75)	<.0001	-7.2655	(85.85)	<.0001
TEGR	-0.3251	(4.09)	0.0432	-0.2961	(3.13)	0.0769	-0.3032	(3.37)	0.0662
FCTD	21.7238	(28.16)	<.0001	30.4341	(43.06)	<.0001	36.5336	(39.66)	<.0001
CDTA	2.0113	(10.08)	0.0015	2.2393	(11.93)	0.0006	2.2501	(12.28)	0.0005
STA	-1.1595	(13.21)	0.0003	-1.4288	(17.84)	<.0001	-1.1460	(11.63)	0.0006
OCFTA	-3.5027	(8.37)	0.0038	-2.9068	(4.91)	0.0267	-3.2728	(6.63)	0.01
SIGMA	16.0805	(15.43)	<.0001	19.7253	(22.69)	<.0001	17.3826	(15.61)	<.0001
METL	-1.5544	(13.46)	0.0002	-1.4285	(11.01)	0.0009	-3.3527	(14.18)	0.0002
Prin1				0.0676	(0.85)	0.3557	0.5164	(3.13)	0.0768
Prin2				0.5400	(30.67)	<.0001	0.5236	(5.99)	0.0144
FCTD*Prin1							-5.7598	(4.56)	0.0328
METL*Prin1							0.5409	(4.26)	0.039
STA*Prin2							-0.3591	(2.53)	0.1114
METL*Prin2							1.2888	(4.8)	0.0284
Model Fit	225.66 ***			260.8 ***			279.3 ***		

Out-of-Sample Forecast Accuracy

Decile	Model I	Model II	Model III
1	64.10	64.10	69.23
2	10.26	10.26	15.38
3	7.69	5.13	0.00
4	2.56	7.69	2.56
5	7.69	5.13	2.56
6	5.13	2.56	2.56
7	0.00	2.56	2.56
8	0.00	0.00	2.56
9	2.56	2.56	2.56
10	0.00	0.00	0.00

- Default probabilities during the test years (2001-2005) are calculated using parameter estimates from 1993-2000 data.
- Firms are grouped into deciles sorted by the forecasted default probabilities in descending order. Then we count the number of defaults in each decile in each year to aggregate over the test years.

Out-of-Sample Forecast Accuracy (AUROC)

Model	Mean	Median	Std. Dev	Min	Max
Model I	0.8256	0.8713	0.1067	0.6758	0.9364
Model II	0.8133	0.8323	0.1163	0.6544	0.9377
Model III	0.8644	0.8681	0.0954	0.7114	0.9669

- AUROC (area under receiver operating characteristic curve) is a widely used forecast accuracy measure in categorical data analysis.
- Predictive power increases as AUROC becomes closer to 1. A value of 0.5 means no predictive power.
- During the test years, on average, Model III (with firm-specific macro exposure) shows the highest AUROC.
- Also, note that Model III shows stable AUROC over the test years.

Concluding Remarks

- The proposed hazard model with firm-specific macro exposures show notably better performance than previous hazard models.

- With full heterogeneity, this model is useful for measuring firm-by-firm default correlations.
  - Produces higher correlations than previous hazard models (companion paper).
  - Implies the importance of modeling firm-specific exposures.